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THE DELIVERY OPTION IN MORTGAGE BACKED SECURITY VALUATION SIMULATIONS

Scott Gregory Chastain
Jian Chen

Fannie Mae
3900 Wisconsin Ave
Washington, DC 20016, U.S.A.

ABSTRACT

A delivery option exists in mortgage-backed security market, which has not been considered in existing mortgage pricing simulation literature. We explain the delivery option in the "To Be Announced" trade. We discuss how the presence of the delivery option effects the use of the standard pricing simulation technique. This technique uses a risk neutral interest rate simulation with a prepayment option model to recover a price which is an expectation over the possible rate outcomes. The simulation technique uses Monte Carlo integration with a suitable selected pseudo or quasi-random sequence. To recover market prices a spread term called the "Option Adjusted Spread" is required. We see that multiple simulations are required to explore the full structure of the delivery option but suggest how to use one simulation to approximate pricing even when the delivery option is present.

1 INTRODUCTION

We will use the abbreviation MBS for mortgage-backed security. We will always be referring to fixed rate agency MBS in this article. The existing literature on mortgage-backed security valuation has focused on the prepayment option (Richard and Roll 1989, Spahr and Sunderman 1992), which is considered the hardest aspect of the problem. To a lesser extent the problem of efficient numerical simulation, in particular the use of low-discrepancy sequences has received some attention (Paskov 1996, Åkesson and Lohoczky 2000).

Practitioners of mortgage-backed security valuation are aware of the need for a spread term to replicate market prices. As is the case in general for fixed income investing, the relative value to other markets is the predominate way of thinking about valuation. In the case of mortgage-backed securities the relative value to the swap market, the treasury market, and the agency bond market all receive attention. Thus the spread will the quoted with respect to some market; for example the LIBOR spread or the treasury spread.

An aspect of the mortgage-backed security market which the practitioners deal with but which has not been described in the context of valuation simulation in the literature is the presence of the delivery option in the agency mortgage-backed security market. This delivery option is commonly refereed to as the TBA trade.

2 THE TO BE ANNOUNCED MARKET

In the excellent description of the mechanics of the Agency MBS market (Tierney 1997) in *The Handbook of Mortgage Backed Securities*, the To Be Announced trade is described as being a trade of a generic pool. The seller announces the exact pool to be delivered two days before settlement. The MBS commitment thus contains an option held by the seller. The buyer specifies only the originating agency, the pass through rate, and the original maturity (usually either 30 years or 15 years). The coupon paid by the mortgage holders is reduced to the pass through rate paid to investors by removal of fees for servicing, administration and credit guaranties. The investor pass through rates are commonly in half integer increments. All other pool characteristics are not specified at the time of the commitment. The seller does not need to own an appropriate MBS at the time of commitment. Since the seller sometimes does not obtain an appropriate MBS for delivery in time for the two day announcement the market has standard procedures for handling these failed deliveries.

3 OPTION ADJUSTED SPREAD

We first describe the valuation method in the context of pricing a particular MBS with known attributes. As described in (Chen 2004), the valuation of a MBS is defined as the expected value of the present value of future cashflows:

$$P = E[V] = E\left[\sum_{t=0}^{M} PV(t)\right] = E\left[\sum_{t=0}^{M} d(t)c(t)\right],$$

where

- P is the price of the MBS,
- V is the value of the MBS, which is a random variable, dependent on the realization of the economic scenario,
- $PV(t)$ is the present value for cash flow at time t,
- $d(t)$ is the discounting factor at time t,
- $c(t)$ is the cash flow at time t,
- M is the maturity of the MBS.

The expectation is taken with respect to a risk neutral measure. An interest rate model is needed for the calculation of the function $d(t)$, and a prepayment model is requirement for the calculation of the path dependent cash-flows, $c(t)$. The cashflow function and in particular the prepayment model depend on knowledge of the MBS attributes. The expectation is approximated using Monte Carlo integration:

$$P = \frac{1}{N}\sum_{p=1}^{N}\left[\sum_{t=0}^{M} d_p(t)c_p(t)\right] \qquad (1)$$

where a subscript is added to indicate that the discount function and the cashflow function are path dependent.

When the interest rate model is calibrated to swap or treasury rate levels and option values and the prepayment model is calibrated to observed prepayment patterns the price returned by equation (1) will be found to not equal the observed market price. A single spread term is added to the equation in the discounted function. This spread, s, is referred to as the option adjusted spread. The discount function

$$\begin{aligned} d(t) &= \prod_{i=0}^{t-1} d(i-1,i) \\ &= \prod_{i=0}^{t-1} \exp(-r(i)\Delta t) \\ &= \exp\{-[\sum_{i=0}^{t-1} r(i)]\Delta t\}, \end{aligned}$$

becomes

$$\begin{aligned} \tilde{d}(t) &= \exp\{-[\sum_{i=0}^{t-1} r(i)+s]\Delta t\} \\ &= \exp\{-[\sum_{i=0}^{t-1} r(i)+s]\Delta t\}\exp\{-st\} \\ &= d(t)e^{-st}. \end{aligned}$$

Then s is solved for such that

$$P_{market} = E\left[\sum_{t=0}^{M} \tilde{d}(t)c(t)\right].$$

With Monte Carlo integration we can rearrange terms to simplify solving for the spread

$$\begin{aligned} E\left[\sum_{t=0}^{M} \tilde{d}(t)c(t)\right] &= \frac{1}{N}\sum_{p=1}^{N}\left[\sum_{t=0}^{M} \tilde{d}_p(t)c_p(t)\right] \\ &= \frac{1}{N}\sum_{p=1}^{N}\left[\sum_{t=0}^{M} d_p(t)e^{-st}c_p(t)\right] \\ &= \sum_{p=1}^{M} e^{-st}\left[\frac{1}{N}\sum_{t=0}^{N} d_p(t)c_p(t)\right]. \end{aligned}$$

We will use the abbreviation OAS for the option adjusted spread.

This is the standard presentation on the OAS approach to MBS valuation. A somewhat different approach is presented in (Kalatay 2005), where the prepayment model is calibrated to market prices instead of being based on observed prepayment patterns.

4 EXAMINING THE COMPONENTS OF OAS

Most people view OAS of MBS as the excess return over a comparable risk-free fixed income investment, generally the LIBOR market, after adjustment of the optionality associated with underlying mortgage loans. Whether LIBOR rates are risk-free rates is still arguable, but it is viewed as the funding cost for most large financial institutions, so OAS can be a measurement of profitability of MBS investment, if financed via the LIBOR market.

Theoretically OAS can be decomposed of several components.

4.1 Credit Spread

Non-Agency MBS have significant credit risk associated with them, so they are generally structured in such a way that different degrees of credit risk are put on different layers of the structured MBS, from B-rated up to AAA-rated. For those non-Agency MBS, credit spread generally consists a big portion of the nominal spread, as well as the OAS. In senior/sub structure, the credit risk comes from the mortgage borrower, and the CMO structure. For a back-end credit enhanced deal, there is additional credit risk coming from the insurance provider. For Agency MBS, the credit component is negligible.

4.2 Liquidity Spread

Agency TBA market is the most liquid MBS market, and generally carry little liquidity premium. However, for very seasoned Agency MBS, and most Non-Agency MBS, liquidity could be an important factor affecting the pricing and OAS numbers. This component is largely determined by market demand and supply. When market panics, the liquidity could play an very important role in determining the price, which is very well demonstrated in the 1998 Russian default event and the subsequent Long Term Capital Management debacle.

4.3 Model Uncertainty Risk Premium

Prepayment model is of vital importance to any MBS-related analysis, and OAS calculation is no exception. Like any econometric forecasting model, prepayment model has forecasting errors, no matter how well it is calibrated. First, there are lots of economic variables, which will affect prepayment behavior, cannot enter the prepayment model, such as local unemployment rate, borrower education level, household income improvement, etc. Second, human behavior is always affected by some non-rational factors, and cannot be fully depicted by any econometric model. And last but not least, people change, so a good model last year might not be a good model this year. Thus OAS could be interpreted as a risk premium to compensate for the model uncertainty. A very good example for this phenomenon is that for very well understood MBS product, like Agency 30-year fixed rate mortgage, generally has low OAS. For a new product with exotic features, e.g., a hybrid adjustable rate mortgage with 40 year amortization term, and 3 year interest only period, and 5 year fixed interest rate period, generally will have a higher OAS.

4.4 Delivery Option

In the TBA market, at the time of commitment, the buyer and seller only need to agree on the issuing agency, note rate, and the seller need to notify the buyer of the exact pool to be delivered two days before the settlement date. So the seller has a very wide range of MBS pools eligible for delivery, and she would naturally deliver the cheapest available MBS pools. So the OAS calculated from TBA price might not represent the TBA universe, but the worst of the universe, and difference of the OAS numbers should be viewed as the option cost for this cheapest-to-deliver option.

4.5 Negative OAS

As we have mentioned above, OAS is generally viewed as the extra return over risk-free rate, so how is it possible be negative? Actually, for recent months, the OAS for prevailing TBA trades have been consistently negative. There could be several reasons for this observation: 1) OAS is the excess return over the life of the MBS. If MBS yield is relatively high, and volatility is low, which has been the case for recent period, people are not very concerned with the MBS optionality in the short term, and as long as they do not hold the MBS till maturity, then can still make profit on short-term trading; 2) There has been large purchase from foreign central banks, and as long as their funding cost remains lower than LIBOR, which is generally the case, and MBS yields higher than alternative investment, which is treasury bond for most of them, buying MBS is still a good investment.

5 VALUATION OF THE TBA TRADE

5.1 Relative Value of the TBA Trade

All descriptions of the OAS method of MBS valuation that we have found in the literature assume that the pool characteristics are known, that is there are defined inputs for the cashflow function. In (Tierney 1997) it is said that the TBA trade can be thought of as a purchase of the average of all similar pools. Putting this statement together with the definition of OAS pricing implies that for the TBA trade one should base the cashflow function on the average characteristics of all deliverable pools. This approach maintains the structure of the OAS pricing formulas.

There are several other possibilities for maintaining the OAS pricing formulas. All that is required is assuming one set of pool characteristics is sufficient for the pricing method. Besides the average of possible pools, one might use the cheapest to deliver characteristics or the most likely to be delivered characteristics. The cheapest to deliver characteristics is based on the assumption the seller has sufficient variety of pools available to deliver that they can select the worst possible pool from the purchasers point of view. On the other hand if the purchasers is actively buying a high volume of securities they may be able to detect delivery patterns and identify the most likely to be delivered.

However, besides using the characteristics for average available pool or the worst available pools in the OAS analysis, we could also calculate the OAS for all representative pools, and assign them a factor loading, and calculate the weighted average OAS as the OAS for TBA trade.

So actually there are three different set of OAS measures could be used:

- OAS for average pool(s), noted as $OAS(\overline{pool})$;
- OAS for cheapest pool(s), noted as $OAS(pool_W)$;
- Average OAS for all available pools $\overline{OAS}(pool)$.

The most flexible and influential characteristic in a TBA trade is the WALA (weighted average loan age), and we would like to examine how different WALA would affect the OAS in a TBA trade. Let's examine the following three TBA trades:

Discount pools, which have price lower than the principal value, because they generally have mortgage rates lower than current rate, either because the mortgage borrower paid points to get a lower rate, or the borrower get the mortgage in a previous period when the mortgage rate is lower then the current mortgage rate. Since the mortgage always pays the principal value at prepayment, the MBS investor benefits from prepayment because she paid a lower than par price, and get the par value back. So in such a trade, seasoned pools should have a higher yield than new pools, and hence a higher OAS.

Current pools, which have price equal to or very close to the principal value. Most likely these mortgage borrowers get the loan in recent period, or from previous periods when mortgage rate are close to today's level. MBS investor should be indifferent to prepayment, as long as the pool keep current. Thus in a flat yield curve environment, the OAS curve with respect to WALA should be pretty flat. And in a upward sloped yield curve, it should be slighted downward sloped, because prepaid principal are discounted lower.

Premium pools, which have price higher than the principal value, because they have mortgage rates higher than current rate, either because the mortgage borrower could only get a high rate mortgage due to imperfect credit, or the borrower get the mortgage in a previous period when the mortgage rate is higher then the current mortgage rate, and she has not refinanced yet. To the opposite of discount pools, the OAS curve should be downward sloped, because MBS investor is hurt by faster prepayment.

5.2 Pricing Specified Pools with TBA OAS

As we have mentioned before, in a TBA trade, the buyer and seller do not need to agree on the specific characteristics of the underlying MBS pools at the time of transaction. However, in some case, the buyer is willing to pay above the TBA price for specified pools, i.e., the current TBA price for 5.5% coupon MBS is \$100, and the buyer want to buy some pools originated in 2002, and there is no market price for this trade. Generally people would use the OAS derived from the TBA market, and price these pools accordingly. A pay-up will be calculated as the difference between the prices of the specified pools and the TBA market. Then there will be some buyer adjustment to make a final offer price. Generally the buyer will not offer the full pay-up for the specified pools, because of the following reasons: 1) They are not sure about using the TBA OAS to price seasoned pools; 2) They require higher OAS to compensate the uncertainty associated with seasoned pools; 3) Seasoned pools have better quality, thus higher OAS.

6 NUMERICAL EXAMPLE

We use the simple prepayment model in Chen(2004) to carry out the Monte Carlo simulation of MBS pricing, in order to acquire the OAS curve for different seasoned pools, given the TBA price.

The following table gives the hypothetical TBA trade prices:

Table 1. TBA Specifications and Prices

TBA	TBA Price
7.5% WAC	100
8.0% WAC	102
8.5% WAC	104
9.0% WAC	106
9.5% WAC	107

We use a simple secant method to calculate the OAS, given the TBA price:

$$OAS_{k+1} = OAS_k - \frac{OAS_k - OAS_{k-1}}{error_k - error_{k-1}} error_k$$

The algorithm proves be very efficient, and can find the optimal solution at error level of 1e-6 within five iterations.

And the following chart gives the OAS curve with respect to WALA for different TBA trades:

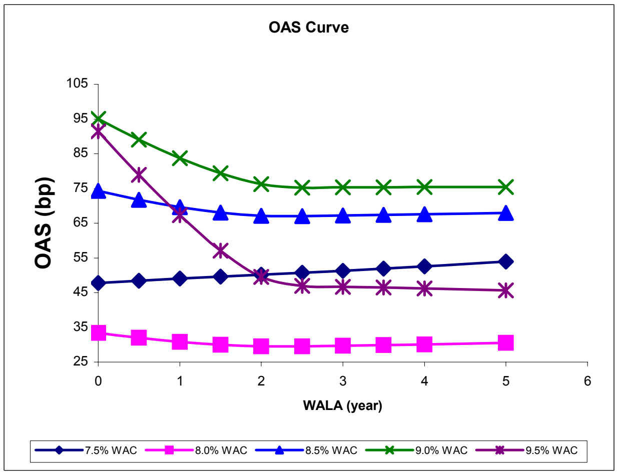

Figure 1. OAS for Different Seasoned MBS

As expected, we see that the OAS curve for discount pools (7.5% WAC) are increasing along with the WALA. As the WAC increases to current coupon (8.0% WAC), the OAS curve is pretty flat with slight downward slope in the beginning. For the premium mortgage pools (9.0% WAC), the OAS curve is obviously decreasing. One interesting phenomenon is for the cuspy coupon mortgage (coupon rate slightly higher than the current coupon, in our example, 8.5% WAC), the OAS actually demonstrated "U"-shaped curve, which can be demonstrated in the following graph.

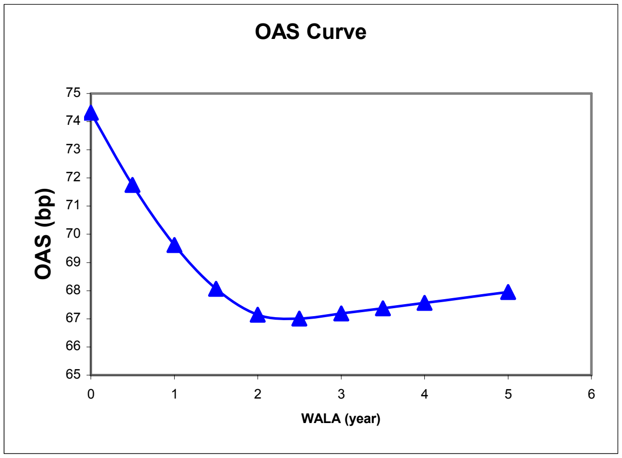

Figure 2. OAS Curve for 8.5% WAC MBS

This can be explained by two factors in the prepayment model: refinance seasoning and burnout effect. The refinance seasoning maxes out at month 30, and after that the burnout effect will slow down the prepayment, so not surprisingly, we see the OAS has the lowest value at month 30, i.e., year 2.5.

7 SIMULATION CONVERGENCE

Our simulation was implemented in the MATLAB 7 programming environment. For the OAS analysis presented above we used the built in MATLAB 7 pseudo-random number generator. MATLAB 7 documentation indicates that this generator is based on the work of Florida State University professor George Marsaglia. In all cases we used uniform random numbers and converted to a normal distribution using the MATLAB language function norminv(). This may be less precise than using the built in MATLAB 7 normal random number generator but it allowed for us to switch uniform number generators without much code change. We used a fixed seed and fixed number of paths across the experiments. The OAS calculation requires repeated calculation of price; that is, repeated Monte Carlo integration. By using a fixed seed and number of paths we believe that bias inherent in the simulation is consistent and we are able to make qualitative observations about the OAS curve.

To understand the convergence properties of our problem we performed experiments on the price function with a fixed OAS in which we varied the seed, the number of paths, and the number generator.

We tried the Mersenne Twister (Matsumoto and Nishimura 1998) pseudo-random number generator in place of the built in MATLAB 7 generator. We used a MATLAB language implementation of the Mersenne Twister written by Peter Perkins who is associated with the maker of MATLAB, The MathWorks. His implementation is derived from the C code published by Matsumoto and Nishimura. For our pseudo-random number tests we used the statistical properties of Monte Carlo integration to find confidence intervals for various choices of number of paths. The confidence intervals obtained with the Mersenne Twister were similar to those obtained with the built in MATLAB 7 generator. The confidence interval is quoted around a single observation so there is more information in the width of the interval than the absolute location of the interval. We use the variance of the path-wise observations in the simulation to estimate the simulations variance at the specified number of paths. This is the variance of repeated trials at that number of paths for true random numbers which can be considered as approximated by repeated trials with a pseudo-random number generator with varying seed.

Table 2. Confidence Intervals with Mersenne Twister

Interval	Number of Paths
[103.9889,105.9426]	100
[104.1827,104.8183]	1,000
[104.5146,104.7167]	10,000
[104.5965,104.6607]	100,000

We also considered the convergence with quasi Monti Carlo. We split the problem into two sets of random dimensions *(X,Y)* in the time basis, which is the natural concept of dimension in this problem. Let X represent the interest rate shocks for the first 40 months on each path and Y represent the remaining months. We used a MATLAB language implementation of the Sobol sequence made available by John Burkardt from Florida State University which produces Sobol vectors in up to 40 dimensions. Thus in our decomposition *(X,Y)* we use Sobol for the X dimensions and Mersenne Twister for the Y dimensions. The use of a low discrepancy sequence means that we can no longer report confidence intervals. The reported results are for a fixed number of paths, 1000, with a varying seed and the same MBS parameters as used for the Mersenne Twister experiment. It is observed that the stability of the price in the 1000 path Sobol experiment exceeds that observed in the confidence interval for 1000 paths for pseudo-random generators.

We have found the build in pseudo-random generator in MATLAB 7 adequate for demonstrating the qualitative aspects of OAS in the presence of the delivery options. Pseudo-random sequences have the advantage of allowing us to work with confidence intervals.

Table 3. 40 Dim Sobol Convergence

Price	Seed
104.5850	200
104.5488	1,000
104.5675	20,000

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AUTHOR BIOGRAPHIES

SCOTT GREGORY CHASTAIN is currently an economist in Fannie Mae. He received his Ph.D. in mathematics at the University of Florida.

JIAN CHEN is currently a financial engineer in Fannie Mae. He received his Ph.D. in computational finance in the Robert H. Smith School of Business, at the University of Maryland. His research interests include simulation and computational finance, particularly with the pricing and hedging for interest rate derivatives and credit derivatives.